A Portion of Profits to Charity: 
Corporate Social Responsibility and Firm Profitability*

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Abstract 
I develop a model in which a firm can choose to donate a portion of its profits to the provision of a public good. Consumers value this public good and are willing to pay a price premium to a firm which makes such a donation. When this price premium is sufficiently large, the firm can raise its net profits by pledging a portion of those profits to provision of the public good. This is more likely when the consumer’s marginal valuation of contributions to the public good is high and when the firm (in the absence of donations) has a high ratio of fixed costs to operating profits. I identify some circumstances under which corporate social responsibility makes consumers worse off. I also analyze firm behavior when giving takes the form of a fixed dollar amount per unit sold. When giving takes this form, firms find it profitable to make a donation to the public good in a smaller range of circumstances compared to when giving is a fixed percentage of profits.

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1. Introduction

Corporations engage in a variety of acts which fall under the term corporate social responsibility (CSR). These can include contributions to charity, actions designed to improve the environment, or a commitment to purchasing their inputs from firms upholding certain labor standards. The question I address in this paper is whether a firm can raise its net profits by donating a percentage of its profits toward the provision of a public good. This could occur if, for example, a pledge by the firm to donate 2% of its gross profits to charity raises gross profits by more than 2%. I show the circumstances under which this can occur in a model in which consumers value the public good provided by the firm’s donation. As a result, consumers are willing to pay a premium to a firm making such a donation.

There are two key determinants of whether a firm would find it profitable to pledge a percentage of its profits to the provision of a public good. The first is the marginal valuation that consumers place on provision of the public good. The higher is this valuation, the more likely it is that the firm will find it profitable to engage in this form of CSR. The second factor is the ratio of fixed costs to operating profit when the firm does not engage in CSR. The higher this ratio, the more likely it is that the firm would want to donate a portion of its profits to provide a public good. When consumers can contribute directly towards the public good via the voluntary contributions mechanism (VCM), I find that the firm makes contributions equal to the noncooperative Nash level of provision found under the VCM in the absence of CSR. In this circumstance, CSR makes consumers worse off relative to the situation where contributions are solely through the VCM, because they finance all of the public good provision via the price premium they pay the firm, but also finance higher profits for the firm.
Donating a percentage of profits to charity is one of many forms that CSR may take. A prominent example is Target, which donates five percent of its profits to charity.\(^1\) It is also fairly common for firms make a fixed charitable donation for each unit of a product sold. In section 4 of the paper, I provide a comparison of these two forms of CSR and show that firms find it profitable to donate a percentage of their profits in a broader range of circumstances than they find it profitable to make a fixed donation per unit sold, as long as the fixed costs of the firm are positive. When fixed costs are zero, the mechanisms are equivalent.

Why are fixed costs important in comparing the performance of these two mechanisms? When giving is fixed per unit sold, contributions towards the public good begin with the very first unit sold. When giving is a fixed percentage of profits, each consumer correctly views herself as having a marginal impact on provision of the public good with her purchase of the private good. However, from the firm’s perspective many of these purchases are inframarginal, because no contributions are made until the firm covers its fixed costs. If fixed costs are zero, the mechanisms are equivalent because, under each, the firm contributes towards the public good from the very first unit sold. Thus, fixed costs play a crucial role in understanding the functioning of the mechanism under which the firm contributes a portion of its profits to charity.

2. Background

There is a large literature on CSR and several theories as to why firms engage in these activities. Recent surveys of this literature are provided by Kitzmueller and Shimshack (2012), Schmitz and Schrader (2015) and Crifo and Forget (2015).\(^2\) One possibility is that firm managers contribute to a public good against the interest of shareholders as a way of satisfying their own preferences.

\(^1\) The five percent figure is stated on the Target website: https://corporate.target.com/corporate-responsibility.

\(^2\) Bénabou and Tirole (2010) provide a general discussion of the motivations for CSR.
This possibility underlies Friedman’s (1970) critique of CSR. If CSR results from an agency problem between the management and stockholders of the firm we should expect to find more CSR in concentrated industries. The reason is that firm rents are higher in concentrated industries and this would give more latitude for management to satisfy their preferences for CSR. By contrast, Baron (2001) models CSR as a response to private politics. In his model, an activist threatens a boycott of the firm unless certain actions (e.g., steps to protect the environment) are taken. Because CSR is undertaken by the firm in order to maximize profits, Baron describes this as strategic CSR. Clearly firms engaged in strategic CSR are not subject to Friedman’s (1970) critique.

Firms may also undertake CSR in order to satisfy the demands of shareholders for the provision of certain public goods. This motivation for CSR is suggested by the analysis of Gordon (2003) and has been developed recently by Morgan and Tumlinson (2012).³ Fernández-Kranz and Santaló (2010: 457) argue that if the public good provided by a CSR firm is a normal good for stockholders, then provision will decrease as the industry becomes more competitive, because this will cause profits to fall. Empirically, Fernández-Kranz and Santaló (2010) find that firms engage in a higher level of CSR when their industry becomes more competitive. They argue that this positive association could result if CSR leads to higher productivity for the firm. By contrast, if CSR is a form of product differentiation, then in more competitive industries they argue we should see a greater variance in CSR levels across firms as they attempt to maximally differentiate themselves.

Fisman et al. (2006), develop a signaling model in which firms engage in acts of corporate philanthropy in order to signal aspects of private good quality which are difficult to observe, even after a product is consumed. The authors cite the example of beef, where the

³ Also see Graff Zivin and Small (2005) and Baron (2006).
conditions under which the beef was raised and the question of whether it was hormone treated cannot be perfectly observed by the consumer even after the beef is consumed. It is less costly for socially-minded firm owners to engage in acts of philanthropy and such owners are also less likely to cut back on unobservable quality. Thus, acts of philanthropy can be an effective signal of product quality. They find that a higher level of market competition is more likely to lead to acts of CSR, because firms in such markets have a greater need for product differentiation.

In Besley and Ghatak (2007), there is a linkage between the purchase of a private good and the firm’s provision of a public good. Consumers understand this linkage, and as a result, are willing to pay a premium to firms which contribute to the public good. Firm contributions take the form of a fixed amount per unit sold. The firms are Bertrand competitors who earn zero profits in equilibrium. Thus the issue of how CSR affects firm profitability is not addressed. Similarly, in Kotchen (2006) firms sell private goods which may be linked to the provision of environmental amenities. Firms again make zero profits as the underlying market structure is perfectly competitive. Another paper along these lines is Bagnoli and Watts (2003). In this paper, consumers obtain a warm glow benefit by making a purchase from a firm which links the purchase of the private good to an increase in provision of the public good. As a result, consumers are willing to pay a premium to firms which link the public and private goods together. Bagnoli and Watts find that an increase in competition may reduce CSR. Their result refers to an increase in competition among firms which are not providing the public good. When these firms charge a high price for the private good, this leads to a substitution towards the firms which are providing the private good which is linked to the public good. Greater competition among this first group of firms reduces this substitution effect and leads to a fall in the provision of the public good.
From a modeling standpoint, the most directly related work is Pecorino (2001).\(^4\) The firm in this paper acts as a non-profit in the sense that it turns 100% of its profits over to the provision of the public good.\(^5\) Consumers understand that their purchase of the private good is linked to the provision of the public good, and this leads them to pay a premium for the private good.\(^6\) In the current paper there is a for-profit firm which endogenously chooses what percentage of its profits to turn over toward provision of the public good. It does this in order to maximize its profits net of the contribution. In the earlier paper, the market for the private good is monopolistically competitive with free entry. Thus, for-profit firms (those not engaged in the provision of the public good) earn zero profits in equilibrium. In the current paper I allow for the possibility that firms may make a profit even absent any contributions towards the public good. This turns out to be an important determinant of whether or not the firm will choose to donate a portion of its profits to charity. Firms are more likely to choose to donate a portion of their profits to the public good if, in the absence of a contribution, they have a high ratio of fixed costs to operating profit. If greater competition lowers operating profit it will be associated with an increase in the use of this form of CSR. However, when giving is fixed per unit sold, I find no relationship between the level of competitiveness and the propensity to donate towards the public good.

One implication of the model is that consumers are willing to pay a premium for private goods whose purchase is linked to a public good. There are number of empirical studies based on

\(^4\) Pecorino (2001) is a partial equilibrium analysis. This analysis was extended to general equilibrium in Pecorino (2010, 2013).

\(^5\) Ghosh and Shankar (2013) develop a model in which consumers obtain a warm glow benefit from giving to the public good and also may obtain an additional benefit if their act of giving becomes public. The firm in their model decides what percentage of the purchase price to devote towards provision of the public good. In contrast to my paper, they find a fairly negative set of results whereby the presence of the firm selling a private good linked to the public good leads to a net decrease in public good provision.

\(^6\) Other related works which share this feature include Posnett and Sandler (1986) who conduct an analysis along these lines in a perfectly competitive market setting. Also of note is the work on lotteries by Morgan (2000) in which purchase of a lottery ticket is linked to provision of a public good, and work by Engers and McManus (2007) on charity auctions.
both experimental data and on field data which are consistent with the idea that consumers are willing to pay such a premium. Included among the studies using field data are Teisl et al. (2002), Casadesus-Masanell et al. (2009), Elfenbein and McManus (2010), and Elfenbein, Fisman and McManus (2012). Hiscox and Smyth (2011), Basu and Hicks (2008) and Arnot, Boxall, and Cash (2006) conduct field experiments which also support the idea that consumers are willing to pay a premium for private goods linked to the provision of a public good.7

3. The Model

I will begin with a short discussion of the properties of the public good, and a brief analysis of contributions towards the public good under the VCM in section 3.1. This analysis provides an important benchmark for understanding the model of CSR developed in the remainder of Section 3. In section 3.2 I will discuss the firm’s output decision taking \( \alpha \), the percentage of profits it donates to charity as given, while in section 3.3, I will endogenize the choice of \( \alpha \). In sections 3.2 and 3.3 I consider a firm’s contributions towards the public good when consumers do not have the opportunity to contribute directly, but in section 3.4, I will also allow for direct contributions by consumers via the VCM.

3.1 The Public Good and the Voluntary Contributions Mechanism

There are \( n \) identical consumers who each enjoy a benefit \( F(G) \) from the public good, where \( G \) is the total contribution towards the public good with \( F'(G) > 0 \). (Prime and double prime superscripts denote, respectively, the first and second derivatives of a function.) In addition, the limit as \( G \) approaches infinity of \( F'(G) \) is less than 1. Later in the paper we consider a special case where \( F''(G) = 0 \), but we otherwise assume \( F''(G) < 0 \). The analysis in this paper is partial

7 Also of note is the empirical work on lotteries. Landry and Price (2007) find that earmarking revenue for education raises expenditures on lotteries. Also see the experimental work of Lange, List and Price (2007).
equilibrium. Among other things, I will ignore any income effects on the demand for either the public good or the private good (which is analyzed in the next section). Thus, for the purpose of this analysis, I am assuming that the marginal utility of income is constant, as would be the case with a quasi-linear utility function. The cost of a contribution is normalized to 1. Under the VCM, the following condition holds at an interior equilibrium:

\[ F'(G) = 1. \]  

(1)

The condition in (1) is completely standard in models of public good provision with quasi-linear utility. Let the value of \( G \) which solves (1) be denoted \( G' \). If \( F'(0) < 1 \), then under the VCM, we will find ourselves at a corner solution with zero contributions.

Later we will compare outcomes under CSR to the outcome under the VCM. If an equilibrium outcome under CSR results in \( F' > 1 \), this implies that less of the public good is provided under CSR compared to the VCM. Similarly, if the equilibrium outcome under CSR results in \( F' < 1 \), then more of the public good is provided under CSR compared to the VCM. These conclusions follow from the assumption that \( F'' < 0 \).

3.2. The Profit Maximizing Level of Output

The question I address is whether a firm can raise its net profits by donating a percentage \( \alpha \) of its gross profits to charity, where \( 0 \leq \alpha \leq 1 \). The choice of \( \alpha \) occurs prior to the choice of output. In this section of the paper, I will take \( \alpha \) as given and analyze the profit maximizing combination of price and output which will be chosen by the firm. The analysis focuses on a single firm. This firm faces the inverse demand curve \( P_0(Q) \), where \( Q \) is the firm’s output and \( dP_0(Q)/dQ < 0 \). The ‘0’ subscript denotes variables associated with the firm when it does not donate to the public good, while the ‘1’ subscript will be used to denote these variables when it does. The underlying
willingness to pay for the private good by consumers is reflected by \( P_0(Q) \), but if the firm devotes a portion of its profits to the public good, consumers will be willing to pay a premium \( k > 0 \) in order to purchase this good. Thus,

\[
P_1 = P_0 + k,
\]

(2)

where \( P_1 \) denotes the price inclusive of the premium. Consumers make their purchases taking the decisions of other consumers as given.

The firm has a fixed cost \( S \) and produces output at the constant marginal cost \( C \). The gross profits of the firm are

\[
\Pi_i = \left( (P_i - C)Q_i - S \right), \quad i = 0, 1.
\]

(3)

Note that the functional dependence of \( P \) on \( Q \) is suppressed in equation (3). The net profits \( \Pi_i^N \) and the profits donated to the public good \( \Pi^G \) are given as follows:

\[
\Pi_i^N = (1 - \alpha)\Pi_i,
\]

(4a)

\[
\Pi^G = \alpha\Pi_i.
\]

(4b)

Thus, the contribution to the public good \( G = \Pi^G = \alpha\Pi_i \).

When the firm sells an additional unit, the increment to gross profits is \( P_1 - C \), where \( \alpha(P_1 - C) \) is donated towards the public good. The consumer making the purchase values this donation at \( \alpha(P_1 - C)F'(\Pi^G) \). \(^8\) Thus, we can write the premium the consumer is willing to pay as follows:

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\(^8\) In the model the consumer correctly anticipates that the firm will turn a positive profit. However, if the firm were to be unprofitable in equilibrium it would contribute nothing to the public good and as a result the consumer would
\[ k = \alpha (P_t - C) F'(\Pi^G). \] (5)

Since \( P_t = P_0 + k \), we can express \( k \) as follows:

\[ k = \frac{\alpha F'(\Pi^G)}{1 - \alpha F'(\Pi^G)} (P_0 - C). \] (6)

Equations (5) and (6) are helpful in understanding how a firm might raise its net profits by pledging to donate a portion of its profits to a public good. The consumer understands that a fraction of the price-cost margin is donated to the public good, and this raises the consumer’s willingness to pay for the private good. This higher willingness to pay implies a higher value of \( P_t \) and this in turn raises the price cost margin leading to an additional increase in the willingness to pay. Thus, there is a feedback effect which amplifies the premium that consumers are willing to pay for a good whose purchase is linked to a donation towards a public good. The feedback effect will not be explosive as long as \( \alpha F' < 1 \). For a sufficiently high value of \( \Pi^G = \alpha \Pi_t \), we will have \( F' < 1 \), and the needed condition will hold.

As shown in part 1 of the appendix, the profit maximizing output \( Q \) of the firm is not a function of \( \alpha \). In other words, a firm donating towards the public good (\( \alpha > 0 \)) produces the same amount of the private good as a firm which does not make a donation (\( \alpha = 0 \)). This implies \( Q_t = Q_0 \). This result is not of independent interest as it matches results in Pecorino (2001, 2010, 2013) in models where firms are nonprofits in the sense that they turn all of their profits over towards not be willing to pay a premium for the private good (i.e., \( k = 0 \)). Also, in the model, there is no distinction between economic profit and accounting profit. In practice, contributions will be a function of accounting profit, not economic profit. As long as the consumer perceives a positive effect on accounting profits from her purchase, she will be willing to pay a premium for the private good when a portion of the resulting profits are used to provide a public good.
provision of the public good (i.e., $\alpha$ is fixed at 1). However, this result will prove helpful in the derivations to follow.

While the level of output is the same regardless of whether or not the firm engages in CSR, as long as $\alpha > 0$ and $F' > 0$, the firm engaged in CSR will charge a price premium such that $P_1 > P_0$. This premium arises because the consumer values the contribution towards the public good. If we substitute for $k$ from (6), the gross profits of the firm may be expressed as follows:

$$
\Pi_1 = \frac{(P_0 - C)Q_0}{(1 - \alpha F'(\alpha \Pi_1))} - S.
$$

(7)

Gross profits appear on both sides of this equation because $\Pi_1$ is in the argument of $F'$. Taking $\alpha$ as given, we can solve for $\Pi_1$ from (7). As shown in part 1 of the appendix, there is a unique value of $\Pi_1$ which satisfies equation (7).

3.3 The Choice of $\alpha$

From the analysis in section 3.2, we know that the choice of $\alpha$ will not affect the firm’s profit maximizing level of output $Q$. However, it will affect $P_1$ because the premium consumers are willing to pay to a firm engaged in CSR is increasing in the percentage of profits donated toward provision of the public good. Taking the derivative of (4a) we have

$$
\frac{d\Pi_1^N}{d\alpha} = -\Pi_1 + (1 - \alpha) \frac{d\Pi_1}{d\alpha}.
$$

(8)

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9 This result is not sensitive to the assumption that $C$ is constant. If $C = C(Q)$ and we replace $C$ by $dC(Q)/dQ$ throughout, we will obtain the same result. See Pecorino (2001: 396). Also, as seen in Pecorino (2010, 2013), this result is not a function of the partial equilibrium setting. The model in these papers is general equilibrium, and competing firms are present, but it is still the case that $Q_1 = Q_0$. In part 2 of the appendix I show that $Q_1 = Q_0$ when competitors are made explicit in a model of Cournot competition.
When $\alpha$ is increased, the firm retains a smaller percentage of the current level of gross profits. This is reflected by the $-\Pi_1$ term in equation (8). On the other hand, the firm captures $(1-\alpha)$ times the increase in gross profits that results from increasing $\alpha$. Gross profits rise because the premium consumers are willing to pay to the firm (in the form of a higher price) is increasing in the percentage of the profits which are turned over to the provision of the public good. Note that

$$d\Pi_1 / d\alpha = (dP_i / d\alpha)Q_0 = (dk / d\alpha)Q_0.$$ From equation (6), we can obtain

$$\frac{dk}{d\alpha}Q_0 = \frac{(P_0 - C)Q_0}{(1 - \alpha F')} \left( F'' + \left[ \frac{\alpha}{1 - \alpha} \right] \Pi_1 F'' \right).$$ (9)

Use (9) plus the definition of $\Pi_1$ in (3) to rewrite (8) as follows:

$$\frac{d\Pi_1^N}{d\alpha} = -[(P - C)Q_0 - S] + \frac{(1 - \alpha)(P_0 - C)Q_0}{(1 - \alpha F')^2} \left( F'' + \left[ \frac{\alpha}{1 - \alpha} \right] \Pi_1 F'' \right).$$ (10)

Before proceeding it is worth noting the following substitution, which will be useful in the analysis which follows:

$$s = \frac{S}{(P_0 - C)Q_0},$$ (11)

where $0 \leq s \leq 1$. The term $s$ measures the fixed costs as a percentage of the operating profits of a firm not engaged in CSR. If firm entry drives down operating profits, then increased competition will be associated with a larger value of $s$. If $s = 1$, then absent a contribution towards the public good, profits are 0. This corresponds to the case of monopolistic competition.

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10 A model of Cournot competition is derived in part 2 of the appendix. If fixed costs are positive, then an increase in the number of Cournot competitors will lead to both a decrease in profitability and an increase in $s$. If fixed costs
If the derivative in (10) is positive at $\alpha = 0$, then it will be optimal for the firm to choose a value of $\alpha > 0$. With $\alpha = 0$, $P_1 = P_0$. Noting this while making use of (11), the condition that $\frac{d\Pi^N}{d\alpha} > 0$ at $\alpha = 0$ may be expressed as follows:

$$F'(0) + s - 1 > 0.$$  \hfill (12)

Note that $F'$ is evaluated at 0, which gives the marginal valuation of the public good at the point where no contributions have been made. The condition in (12) is sufficient to ensure that there exist positive values of $\alpha$ that raise the firm’s profits relative to $\Pi_0$ (i.e., profits when it does not engage in CSR). As shown part 1 of the appendix, this condition is also necessary. That is, if the condition in (12) does not hold, then the firm will choose a corner solution with $\alpha = 0$.

When $F'(0) > 1$, the condition in (12) will hold and the firm will choose a positive value of $\alpha$. This is the same condition needed to ensure an interior equilibrium under the VCM and it implies that at a zero level of provision, the marginal valuation of a one dollar contribution towards the public good is greater than a dollar. However, because $s \geq 0$, there are conditions under which a firm would donate to the public good when the VCM results in a contribution of 0. This occurs when $F'(0) < 1$, but $F'(0) + s > 1$. The higher the value of $s$, the more likely it is for the condition in (12) to be met. If a greater level of competition is associated with a higher value of $s$, then it will also be associated with a greater prevalence of CSR because (12) is more likely to hold. In this sense, the model is consistent with Fernández-Kranz and Santaló’s (2010) empirical finding that an increase in industry competition leads to an increase in CSR. Under monopolistic competition, $s = 1$ and (12) holds for all $F'(0) > 0$. This is summarized as Result 1:

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are zero, an increase in the number of Cournot competitors will cause profits to decrease, but $s$ will remain constant at zero.
**Result 1:** (i) A necessary and sufficient condition for the firm to choose a value of $\alpha > 0$ is $F'(0) + s - 1 > 0$. (ii) Since this condition holds whenever $F'(0) > 1$, the firm always contributes to the public good when individuals would contribute under the VCM, and sometimes contributes when contributions under the VCM are 0. (iii) The needed condition is more likely to hold for a higher value of $s$, which may be associated with a an increased level of competition within an industry.

Why is the condition more likely to hold for higher values of $s$? In choosing a positive value of $\alpha$, the firm is giving away a part of its profits. If the firm is highly profitable in the absence of CSR, then donating a given percentage of profits is more costly to the firm than if it is not very profitable in the absence of CSR. When $s = 1$, the condition holds for all $F' > 0$, because the firm initially has zero profits and therefore nothing to lose by promising to donate a portion of its profits to the provision of the public good. This can be seen clearly from (8). At $\alpha = 0$,
$$d\Pi_1^n / d\alpha = -\Pi_0 + (d\Pi_1 / d\alpha).$$
As fixed cost $S$ rises, $\Pi_0$ shrinks, making it more likely this derivative is positive at $\alpha = 0$. If $\Pi_0 = 0$, we are guaranteed the needed condition will hold (given $F' > 0$) so that the firm will choose $\alpha > 0$.

When fixed costs $S = 0$, we have $s = 0$, and the firm contributes towards the public good under exactly the same circumstances ($F'(0) > 1$) as individuals operating under the VCM. Why do fixed costs play such an important role in how this mechanism functions? All consumers make their purchase decisions simultaneously. In equilibrium each consumer correctly views herself as having a marginal impact on both profits and contributions towards the public good. Thus, all consumers are willing to pay a premium $k > 0$ on each purchase of the private good. However, the firm does not contribute a penny to provision of the public good until operating
profits exceed fixed cost. Thus from the firm’s perspective, many purchases are inframarginal in the sense that they do not trigger a contribution. By contrast, when fixed costs are $0, the firm makes a contribution from the first unit sold. Thus, a high ratio of fixed costs to operating profits in the absence of a contribution (i.e., a high value of $s$) makes contributing a portion of profits towards the public good a more attractive strategy from the perspective of the firm.

To see how this works, consider a numerical example in which fixed costs $S = 200,000$, $Q_0 = 500$, and $P_0 - C = 500.11$ The marginal valuation of the public good $F' = 0.8$ is a constant.

When the firm sets $\alpha = 0$ it earns net profit of $50,000. When it raises the contribution to $\alpha = 0.01$ (1% of gross profits), consumers are willing to pay a premium of approximately $4.03 for each unit sold. (This may be computed from (6).) As a result, the net profit of the firm rises to approximately $51,495, which is an increase of $1,495 relative to when $\alpha = 0.12$ Each consumer correctly perceives that if she buys one less unit of the private good, gross profits of the firm will fall by about $504.03$. Contributions to the public good will fall by $5.04$, where the consumer values these contributions at $0.8(5.04) = 4.03$. Thus, the price premium paid by each consumer is fully justified by the marginal effect of her purchase on contributions towards the public good.

As a result, all 500 consumers pay the price premium of $4.03 to purchase the private good. However, from the firm’s perspective it is not unit the 397th unit is sold that fixed costs are covered and contributions towards the public begin.13 Thus, even though consumers only value contributions at 80 cents on the dollar, the firm can raise net profits by making contributions towards the public good. One hundred percent of consumers pay a price premium, but in this example, only about 20% of purchases trigger a contribution.

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11 These solutions are obtained with the demand function, $Q = 1200 - P$ and marginal cost $C = 200.$
12 Gross profits are approximately $(504.03)(500) - 200,000 = 52,015$. With $\alpha = 0.01$, net profits are 0.99 times gross profits.
13 Each unit sold raises firm profits by $504.03$. Divide $200,000$ by $504.03$ to see that positive contributions begin with the $397$th unit sold.
Having analyzed the necessary and sufficient conditions for a firm to make a positive contribution, the next step is to solve the model assuming that this condition has been met. To do so, set the derivative (10) equal to 0, which leads to the following:\textsuperscript{14}

\[ F'' = 1 - \alpha \Pi_1 F'' - s(1 - \alpha F')^2. \] \hfill (13)

Note that \( F' \) and \( F'' \) are both functions of \( \alpha \Pi_1 \). Using (7) and (13), it is possible to solve for \( \alpha \) and \( \Pi_1 \).\textsuperscript{15} Under the VCM, individuals would contribute until \( F' = 1 \). The middle term on the right-hand side of (13) is positive, while the third term is negative. Thus it is uncertain whether \( F' \) will be greater or less than 1 when contributions are made by a firm. As a result, we cannot be certain as to whether contributions by the firm are greater than or less than contributions under the VCM. If \( s = 0 \) (i.e., fixed costs \( S = 0 \)), then from (13) we will have \( F' > 1 \) implying that contributions will be less than under the VCM. Thus, we can see at least one special case in which firm giving results in lower contributions than the VCM. We know from Result 1 that when \( F'(0) < 1 \) and \( F'(0) + s > 1 \), contributions are positive under firm giving but 0 under the VCM. This constitutes a set of cases in which giving by the firm exceeds giving under the VCM. Thus, in general it is not possible to say whether giving by the firm is more or less than giving under the VCM.

We know from previous work with not-for-profit firms (e.g., Pecorino 2001) that a firm linking the provision of the public good to the private good will provide a greater amount of the public good than is achieved under the VCM.\textsuperscript{16} Mathematically what may prevent this in the current model is the \( F'' \) term in equation (13). The for-profit firm understands that when \( \alpha \) is

\textsuperscript{14} The derivation makes use of the substitutions from equations (2), (6), and (11).

\textsuperscript{15} While there is a unique mapping from \( \alpha \) to \( \Pi_1 \), it is possible that multiple value of \( \alpha \) solve equations (7) and (13). If there are multiple local maxima, the firm will simply choose the value of \( \alpha \) associated with the highest net profits.

\textsuperscript{16} Morgan (2000) obtains the same result under the lottery mechanism.
increased, the contribution towards the public good \( I^G \) rises, and the marginal valuation of the contribution \( F' \) falls. This reduces the per unit premium \( k \) that consumers are willing to pay for the private good. Because the firm has (in this section of the paper) monopoly power over its provision, recognition of this effect causes the firm to provide less of the public good. As a result we cannot say in general whether or not public good provision by the firm will be greater or less than the amount achieved under the VCM.

To gain some additional insight, it will be helpful to consider a special case. Suppose that benefits from the public good are linear in contributions so that \( F' \) is a constant and \( F'' = 0 \). Further, suppose that \( F' < 1 \).\(^{17}\) Under these assumptions, (13) forms a quadratic equation in \( \alpha \).

Solving this equation yields\(^{18}\)

\[
\alpha = \left(1 - \frac{1 - F'}{s}\right)\left(\frac{1}{F'}\right). \tag{14}
\]

It can be seen from (14) that the condition \( F' + s - 1 > 0 \) is required in order to have \( \alpha > 0 \). By taking the appropriate derivatives of (14), it can be shown that \( \alpha \) is increasing in both \( F' \) and \( s \). Thus, a higher percentage of profits are donated when these contributions provide higher benefits to the consumer and when the firm has a higher ratio of fixed costs to operating profits. This is summarized as Result 2:

**Result 2:** When the benefit provided by the public good is linear in contributions, with \( F' < 1 \), the solution for \( \alpha \) is given by (14). The percentage of profits contributed towards the

\(^{17}\) This is a necessary restriction for the linear case. If \( F' > 1 \) and is constant, then from (7) the firm can drive its gross profits to infinity by an appropriate choice of \( \alpha < 1 \). As long as \( \alpha < 1 \) this will imply infinite net profits as well. This can result, because a donation of a dollar to the public good produces more than a dollar of benefit for the consumer. As a result, there is (with an appropriate choice of \( \alpha \)) an explosive feedback effect between donations to the public good and the consumer’s willingness to pay.

\(^{18}\) The positive root of the quadratic equation yields a value of \( \alpha > 1 \), which is clearly not feasible.
public good is increasing in the marginal valuation of a contribution $F'$ and the ratio of fixed costs to operating profits $s$.

Note that when $F' < 1$, contributions under the VCM are zero. Thus, in this special case, the firm’s contribution is greater than under the VCM, but this result does not hold in general. Even if a firm does provide more of the public good than under the VCM, there is never a presumption that the firm would supply the socially optimal level.

In the model, competition from other firms is implicit and is reflected in $\Pi_0$, which is the level of profits the firm would earn if it did not engage in CSR. In part 2 of the appendix, I analyze a model where the firm is embedded in a model of Cournot competition and show that the results of this section continue to hold. Thus, making the firm’s competitors explicit does not affect the results. The analysis in the appendix assumes that the other firms do not engage in CSR, but if they do undertake CSR this also has no effect on the model outcomes as long as they contribute to a public good which is different from the public good contributed to by the firm we analyze here. Moreover, these firms have a strict incentive to contribute towards a public good that no other firm is contributing to in order to avoid direct competition in this activity. Suppose, however, that it is not possible for each firm to contribute towards a distinct public good so that two or more firms are potential contributors towards the same public good. If $F'(0) + s - 1 > 0$, it cannot be an equilibrium for all firms to refrain from making a contribution towards the public good. Thus, when this condition holds, equilibrium will require one or more firms making a contribution towards the public good.19

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19 Pecorino (2010, 2013) considers multiple firms contributing towards the same public good, though the firms in these models are non-profits in the sense that they contribute all their profits to the provision of the public good. When an additional firm decides to contribute towards the public good, the profits of each firm goes down, but joint profits (and therefore joint contributions towards the public good) go up. In the current model, the previously discussed monopoly power over the provision of the public good would be weakened if another firm were to
3.4. The Model when Individuals are Allowed to Contribute through the VCM

Thus far it has been assumed that individuals are not allowed to make a direct contribution towards the public good via the VCM. This could correspond to a situation in which the firm faces lower transaction costs in making a contribution than individuals do. Also, if \( F'(0) < 1 \), contributions under the VCM are 0, but the firm will make positive contributions as long as \( F'(0) + s \cdot I > 0 \). Even if \( F'(0) > 1 \), it is possible that the firm will contribute a sufficient amount towards the public good so that in equilibrium \( F' < 1 \). Again, in this circumstance, individuals will make a zero contribution under the VCM.

For individual contributions to possibly be relevant, it must be the case that the equilibrium described in Section 3.3 results in \( F' > 1 \). This will be the case if the solution to the model in Section 3.3 yields \( I\tilde{F} < G' \) so that contributions are lower than when only the VCM is utilized. Let’s assume that this is true. Individuals would then like to make a contribution via the VCM if this is possible. As before assume that \( \alpha \) is set prior to the consumer decision to purchase from the firm, but that consumer decisions to buy from the firm and individual decisions to contribute under the VCM occur simultaneously. In any equilibrium in which consumers contribute under the VCM, we must have \( F' = I \), where the argument of \( F' \) would be the sum of contributions by the firm and contributions under the VCM. This changes the optimization problem of the firm in an important way. The choice of \( \alpha \) determines \( I\tilde{F} \), but for \( I\tilde{F} \leq G' \), the firm can optimize while taking \( F' \) as constant at 1. Over this range, we will have \( F'' = 0 \). For values of \( \alpha \) such that \( I\tilde{F} > G' \), the firm’s problem reverts back to the analysis from Section 3.3.

contribute towards the public good. This strongly suggests that total contributions would rise, but the profits to each firm from engaging in CSR should be lower.
When the marginal valuation of a contribution toward the public good $F'$ is constant, the marginal effect on net profits from increasing the percentage of profits contributed, $d\Pi_i^N / d\alpha$, is always positive if $s > 0$. Thus when $s > 0$, the firm will choose $\alpha$ such that its contributions towards the public good $\Pi_i^F = G_i^V$, their value under the VCM. This implies that individuals make zero contributions towards the public good in equilibrium. However, the possibility that individuals could contribute directly alters the equilibrium outcome and raises the level of public good provision relative to a situation in which individuals are unable to contribute. This occurs because the monopoly power the firm has over the provision of the public good is broken by the potential contributions under the VCM. Note that in this circumstance, consumers are made worse off by the presence of CSR compared to when contributions occur through the VCM only. They obtain the same amount of the public good as under the VCM and they finance this through the price premiums they pay the firm for the private good it provides. However, these price premiums also finance higher profits for the firm, implying that consumers must be worse off on net. If a dollar of income to consumers is weighed equally in social welfare as a dollar of profits of the firm, this is simply a transfer between consumers and the firm which does not reduce social welfare relative to a situation in which CSR is not undertaken.

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20 To verify this, use (2), (6) and (11) in (10) with $F' = 1$ and $F'' = 0$, to see that $d\Pi_i^N / d\alpha > 0$ if $s > 0$ and $d\Pi_i^N / d\alpha = 0$ if $s = 0$.

21 The analysis in the main body presumes that there is a unique value of $\alpha$ that solves the model in Section 3.3. If there are instead multiple local maxima the equilibrium contribution of the firm will at least equal $G_i^V$, but could be higher if there is a local maxima for a value of $\alpha$ such that $\Pi_i^F > G_i^V$, where the realized net profits of the firm are greater than when $\alpha$ is chosen such that $\Pi_i^F = G_i^V$. This seems sufficiently unlikely that I am relegating discussion to a footnote, but I do not believe that this case can be ruled out. If we allow for this case, then the result below should read that public good provision by the firm is greater than or equal to $G_i^V$. If $\Pi_i^F > G_i^V$, consumers might possibly benefit from the presence of CSR.

22 Remember that for the purpose of this analysis we are presumed to be in a case in which the firm contributes less than $G_i^V$ when contributions under the VCM are not possible. If firm contributions exceed $G_i^V$ when contributions under the VCM are not possible, allowing for contributions under the VCM has no effect on the model.
How can CSR make consumers worse off relative to when contributions only occur through the VCM? As discussed in Section 3.3, while each consumer correctly views herself as having a marginal impact on provision of the public good when she makes a purchase of the private good, from the firm’s perspective some (perhaps many) of these purchases are inframarginal in the sense that they do not trigger a contribution towards the public good. These inframarginal purchases are the source of the gain for the firm and the loss for consumers.

The analysis above is summarized as follows:

**Result 3:** Suppose in the absence of individual contributions under the VCM, the firm would choose $\alpha$ such that $\Pi^G < G^V$ and that $s > 0$. If individuals are then allowed to contribute under the VCM: (i) The firm will choose $\alpha$ such that $\Pi^G = G^V$. Thus, no contributions will be made under the VCM in equilibrium. (ii) The firm’s net profits will increase relative to $\alpha = 0$. (iii) Consumers are worse off in this equilibrium than they would be if the firm did not engage in CSR and all contributions were via the VCM. (iv) Social welfare as measured by the sum of consumer and producer surplus is the same as would be experienced under the VCM.

Note that if $s = 0$, $d\Pi^V_i / d\alpha = 0$. Thus, the firm cannot raise its net profits by undertaking CSR when both $s = 0$ and consumers can contribute directly to the public good via the VCM. Since net profits do not change when $\alpha$ is increased, the firm is indifferent among choices of $\alpha$ which lead to contributions $\Pi^G$ such that $0 \leq \Pi^G \leq G^V$. As a result, there is an indeterminacy over the firm’s choice of $\alpha$ and its resulting level of contribution towards the public good. However total contributions, inclusive of those made by individuals through the VCM.

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23 See footnote 20.
VCM, will sum to \( G^U \). Since net profits of the firm do not rise with \( \alpha \) over the relevant range, all premiums paid by consumers are contributed towards the public good, and consumer welfare is the same as when only the VCM is utilized. We obtain this same result in the next section for any \( s \geq 0 \) when we consider contributions which are fixed per unit sold by the firm and contribution under the VCM are allowed.

4. The Form of the Contribution

Donating a percentage of firm profits is just one way that a firm might donate to a public good. One alternative is simply to donate a fixed dollar amount for each unit sold. Once again, assume that consumers cannot contribute directly towards the public good via the VCM. Let \( x \) be the per unit donation and let \( \Pi^N_x \) be the net profits of the firm. We have

\[
\Pi^N_x = \left( (P_x - C - x)Q_x - S \right).
\] (15)

As with the earlier model, \( P_x = P_0 + k_x \), where a ‘0’ subscript is used to denote variables for a firm which does not make a donation. Each time a consumer purchases the private good, she knows that \( x \) dollars are donated to the public good producing a benefit of \( xF'(xQ_x) \). Thus,

\[
k_x = xF'(xQ_x).
\] (16)

How does the premium in (16) compare the premium in (5) which is derived assuming that a percentage of profits are donated to charity? If the argument of \( F \) is the same so that \( F' \) is equal across the two cases, then the two premiums will be equal if we set \( x = \alpha(P_1 - C) \). Thus, in terms of the premium generated from the consumer, there can be an equivalency between the two methods of engaging in CSR. What we shall see below, however, is that the necessary and
sufficient condition for the firm choosing to set \( x > 0 \) is more restrictive than the corresponding condition for setting \( \alpha > 0 \).

Substitute for \( P_x \), and take the partial derivatives of \( \Pi^N_x \) with respect to \( Q_x \), and \( x \) to obtain

\[
\frac{\partial \Pi^N_x}{\partial Q} = P_0 + Q_x \frac{dP_0}{dQ_x} - C + xF''+x^2Q_xF''-x = 0, \tag{17}
\]

\[
\frac{\partial \Pi^N_x}{\partial x} = F''+xF_xF''-1 \leq 0. \tag{18}
\]

As shown in part 1 of the appendix, \( Q_x = Q_0 \). Once again, the act of contributing towards the public good does not affect the firm’s choice of output for the private good.

A necessary and sufficient condition for the firm to choose \( x > 0 \) is that the derivative in (18) be positive at \( x = 0 \). This requires

\[
F'(0) - 1 > 0. \tag{19}
\]

In contrast to our earlier result, the condition in (19) is independent of \( s \) and depends only on the shape of the benefit function for the public good \( F \). Thus, in contrast with the findings of Fernández-Kranz and Santaló (2010), when contributions are fixed per unit sold, there is no relationship between market competition and the use of CSR by the firm. The condition in (19) is more restrictive than the condition in (12) implying that there are a narrower range of circumstances in which a firm will find it worthwhile to make a fixed dollar contribution vs. contributing a fixed percentage of its profits. The condition in (19) is the same as the necessary...
and sufficient condition for a contribution to be made under the VCM. When (18) holds as an
equality we have the following:

\[ F' = 1 - xQ_o F'' > 1. \] (20)

Since \( F' > 1 \), the level of provision by the firm is lower than would occur under the VCM (under
which \( F' = 1 \)). When only the firm in question has the ability to contribute to the public good, it
has monopoly power in this dimension, where this power is reflected by the \( F'' \) term in equation
(20). This term reflects the reduction in the premium \( k_x \) that occurs when the firm raises its
contribution \( x \). The analysis above is summarized as follows:

**Result 4:** When giving takes the form of a fixed dollar amount per purchase: (i) The
necessary and sufficient condition for a positive level of contributions is the same as
under the VCM and is more restrictive (i.e., contributions are made in a smaller range of
circumstances) then when giving takes the form of a fixed percentage of profits. (ii)
When the firm makes a positive contribution, it will be at a lower level than would occur
under the VCM.

A firm considering a fixed dollar contribution is less likely to engage in CSR compared
with a firm contemplating a donation which is a fixed percentage of their profits. Conditional on
a contribution occurring, what can we say about the amount of the contribution under these two
forms of CSR? As demonstrated in the appendix part 1, we have the following result:

**Result 5:** Assume that \( F'(0) > 1 \) and that \( x \) and \( \alpha \) are chosen optimally. (i) When fixed
costs are zero, contributions and net profits are equal under the two mechanisms. (ii)
When fixed costs are positive, contributions and net profits are both higher when giving
takes the form of a percentage of profits than when it takes the form of a fixed contribution per unit sold.

If fixed costs are positive, then contributions and firm profits are both higher when the firm optimally chooses a percentage of profits to give to charity compared to a firm which optimally chooses a fixed contribution per unit sold. This assumes that we are not at a corner solution with zero contributions in either model (i.e., \( F'(0) > 1 \)). Results 4 and 5 demonstrate that fixed giving per unit sold fares poorly relative to giving a percentage of firm profits. This mechanism is used in a smaller range of circumstances and leads to a lower level of public good provision and a lower level of net profits for the firm when it is utilized. Under both mechanisms, a price premium is paid by consumers on every unit sold. However, when giving is a percentage of profits, the firm does not contribute until fixed costs have been covered. In other words, some (perhaps many) purchases by consumers are inframarginal from the firm’s perspective, because they do not trigger a contribution. When giving is fixed per unit sold, the firm must contribute from the very first unit which is sold. Thus, fixed giving per unit sold is less attractive from the firm’s perspective.\(^{24}\) As a result, the firm gives in a wider range of circumstances and gives a greater amount when contributions are a percentage of profits. One way to see the role that fixed costs play in explaining the difference between the two mechanisms is to realize that when fixed costs are zero, the mechanisms are equivalent.

Suppose that firms give a fixed amount per unit sold, but that consumers may also contribute via the VCM. From (20), \( F' > 1 \) so under the equilibrium of this model individuals

\(^{24}\) By an analogous set of arguments, giving a percentage of profits leads to more public good provision than does giving a fixed percentage of the firm’s revenue. Under this alternative, the expression for the price premium is the same as in (6). Under either method of giving, the premium \( k \) is paid on every unit sold. When giving is a percentage of profits, the firm does not make a contribution until fixed costs are covered. By contrast, when giving is a percentage of revenue, giving begins with the first dollar of revenue. As a result, this version of the model produces a set of results analogous to those found when giving is fixed per unit sold.
would always want to make a contribution under the VCM if possible. In any equilibrium in which these individuals contribute, we must have $F' = 1$. Once again, in choosing $x$, the firm can take $F'$ as constant at 1 as long as $0 \leq xQ_0 \leq G^y$. This implies $F'' = 0$ over the relevant range. From (18), we can see that net profits of the firm will not vary with $x$ over this range. Thus, when the firm’s contribution towards the public good is fixed per unit sold and consumers can contribute directly towards the public good, the firm cannot raise its net profits from CSR. In this case, the firm is indifferent among choices of $x$ that result in values of $xQ_0$ between 0 and $G^y$. However the sum of the contributions by the firm and individuals will be $G^y$. Since net profits of the firm do not rise, all the premiums paid by consumers are contributed towards the public good. This leaves consumers indifferent between the outcome under CSR and the outcome under the VCM alone. This is the same result we obtained in Section 3.4 when giving is a percentage of profits and $s = 0$.

From the firm’s perspective fixed contributions per unit sold fare poorly when compared to giving as a percentage of profits, but this is a commonly observed method of CSR. There may be practical advantages of this method which are not captured by the analysis of this paper. In particular, a fixed contribution per unit sold may be more transparent from the consumer’s perspective and this may make consumers particularly responsive to this form of giving.

5. Conclusion

In this paper, I identify conditions under which a firm can raise its net profits by pledging to donate a percentage of its profits to the provision of a public good. This can occur because consumers understand the linkage between the purchase of the private good and the provision of a public good which they value. One key determinant of whether a firm would like to contribute
to the public good is the ratio of fixed costs to operating profit in the absence of a contribution. When this ratio is high, a firm is more likely to want to contribute a positive percentage of its profits to the public good. If an increase in competition raises the ratio of fixed costs to operating profit, then it will tend to promote CSR. This implication of the model has been supported in empirical work by Fernández-Kranz and Santaló (2010).

When consumers can contribute towards the public good directly via the VCM, the firm provides the same amount of the public good that would be provided under the VCM in the absence of CSR. Consumers are made worse off by CSR in this circumstance, because they obtain the same level of public good provision as under the VCM, but they pay more. They finance the entire level of provision via the price premiums they are willing to pay to the firm for the private good, but they also finance the higher level of firm profits which are obtained via CSR.

I also compare the outcomes when a firm gives a percentage of its profits to the public good to a mechanism under which it donates a fixed sum per unit sold. I find that there are a narrower range of circumstances under which a firm would find it profitable to donate a fixed sum per unit sold. In addition, contributions given as a percentage of profits are always greater than or equal to contributions given on a per unit basis. Contributions are equal only if fixed costs are zero or if we are operating at a corner solution with zero contributions under both mechanisms. Otherwise, contributions are strictly higher when giving is a percentage of profits. Under the same conditions (no corner solution, positive fixed costs) net profits of the firm are also strictly higher when giving is a percentage of profits.

Acts which might be considered corporate social responsibility are consistent with profit maximization by the firm. It is not obvious that a firm donating a portion of its profits to charity
could actually raise its net profits in the process, but the model presented here shows how this can occur. The key assumptions are that consumers value the public good and that they understand the connection between their own purchase and an increase in provision of the public good. These assumptions been employed by a number of papers in the literature on corporate social responsibility and if they hold, the model suggests a fairly wide range of circumstances under which firms can make money by giving money away.
Appendix

Part 1 – Proofs

Part 1 of the Appendix will consist of proofs of several assertions made in the main body of the paper.

1. $Q_1 = Q_0$

This first proof is to establish that output of the private good is the same regardless of whether or not the firm chooses $\alpha > 0$. For the firm which does not engage in CSR, the first order condition from profit maximization implies

$$\frac{d\Pi_0}{dQ} = P_0 + Q_0 \frac{dP_0}{dQ_0} - C = 0,$$

(A.1)

where $P_0$ and $Q_0$ are the profit maximizing price and quantity chosen by this firm. Now consider a firm with $\alpha > 0$. The first order condition to its profit maximization problem implies

$$\frac{d\Pi_1^N}{dQ} = (1 - \alpha)\left(P_1 + Q_1 \frac{dP_1}{dQ_1} - C\right) = 0.$$

(A.2)

Using (2) and (5), note the following (where the arguments of $F$ are suppressed):\(^{25}\)

$$\frac{dP}{dQ_1} = \frac{dP_0}{dQ_1} + \frac{dk}{dQ_1},$$

$$\frac{dk}{dQ_1} = \alpha F'' \frac{dP_1}{dQ_1} + \frac{\alpha^2 (P_1 - C)}{(1 - \alpha)} F'' \frac{d\Pi_1^N}{dQ}.$$

---

\(^{25}\) Public good provision is a function of the donated profits, but using (4a) and (4b), we can write $\Pi^0 = \alpha \Pi_1^N / (1 - \alpha)$. Thus, we can write $F\left(\alpha \Pi_1^N / (1 - \alpha)\right)$. When deriving the expressions below, it is helpful to express the argument of $F$ in this manner.
Since $d\Pi^N_i / dQ = 0$ by the first order condition in (A.2), the two equations above can be combined to yield $dP_i / dQ_i = (dP_0 / dQ_i)(1 - \alpha F'')$. Use this plus the solution for $k$ in (6) to rewrite (A.2) as follows:

$$\frac{d\Pi^N_i}{dQ} = \left( \frac{1 - \alpha}{1 - \alpha F''} \right) \left[ P_0 + Q_1 \frac{dP_0}{dQ_i} - C \right] = 0.$$  
(A.3)

We know from (A.1) that if we set $Q_1 = Q_0$, that the term in brackets will equal 0, and equation (A.3) will be satisfied. Thus, a firm donating a portion of its profits to charity will produce the same level of output as a firm which chooses not to do so.

2. The value of $\Pi_i$ which solves (7) is unique

For convenience, I will reproduce (7) here:

$$\Pi_i = \frac{(P_2 - C)Q_0}{(1 - \alpha F''(\alpha \Pi_i))} - S.$$  
(7)

First, if $F'$ is constant (with $\alpha F' < 1$), then we have a direct unique solution for $\Pi_i$. Now consider the case where $F'' < 0$. Suppose first that when $\Pi_i = \Pi_0$, $\alpha F''(\alpha \Pi_0) > 1$. For a sufficiently large value of $\alpha \Pi_i$, $F' < 1$. Thus, there exists some value of $\Pi_i > \Pi_0$ at which we will have $\alpha F''(\alpha \Pi_i) = 1$. At this point, the left hand side of (7) is finite and the right-hand side is infinite. Further increases in $\Pi_i$ will cause the left-hand side to monotonically increase and the right-hand side to monotonically decrease.\(^{26}\) Thus, there will be one and only one value of $\Pi_i$ such that the two sides are equal. Lastly, suppose that when $\Pi_i = \Pi_0$, $\alpha F''(\alpha \Pi_0) < 1$. At $\Pi_i = \Pi_0$, the right-hand side of (7) exceeds the left-hand side. Since the left-hand side is monotonically increasing in $\Pi_i$ and the right-hand side is monotonically decreasing in $\Pi_i$, there

\(^{26}\) Since $F'' < 0$, increases in $\Pi_i$ cause $F'$ to decrease.
will again be a unique value of $\Pi_I$ which satisfies (7). Since $\Pi_I$ is uniquely determined, so are $\Pi_I^y$ and $I^F$.

3. The Necessary Condition for a Positive Contribution (Result 1, part (i))

From equation (12), the condition $F'(0) + s - l > 0$ is sufficient to ensure that the firm chooses $\alpha > 0$, because it guarantees that profits are increasing in $\alpha$ at $\alpha = 0$. The purpose of this part of the appendix is to establish (as stated in Result 1) that the condition $F'(0) + s - l > 0$ is also necessary for a firm to choose $\alpha > 0$. The nature of the demonstration is to show that if profits at $\alpha = 0$ are higher than when $\alpha$ takes on a positive value in the neighborhood of 0, then it must necessarily be true that profits at $\alpha = 0$ are higher than profits at all positive values of $\alpha$.

For any value of $\alpha > 0$, firm profits are higher than at $\alpha = 0$ if

$$
(1-\alpha)((P_0 + k - C)Q_0 - S) > (P_0 - C)Q_0 - S.
$$

(A.4)

Substitute for $k$ from (6) and rearrange to get

$$
\frac{(1-\alpha)(P_0 - C)Q_0 F'(\alpha \Pi_I)}{1-\alpha F''(\alpha \Pi_I)} > (P_0 - C)Q_0 - S.
$$

(A.5)

Note that the argument of $F'$ has been included in the expression above. Divide both sides by $(P_0 - C)Q_0$, use (11) and rearrange to get

$$
\alpha < \frac{F'(\alpha \Pi_I) + s - 1}{s F'(\alpha \Pi_I)}.
$$

(A.6)

The right-hand side of (A.6) sets up a cutoff value of $\alpha$. If $\alpha$ is below this value, profits for the firm are higher than at $\alpha = 0$. If $F'(0) + s - 1 < 0$, then an increase in $\alpha$ above 0 lowers firm
profits, i.e., there are no positive values of $\alpha$ in the neighborhood of 0 which satisfy the condition in (A.6). If $F'$ is constant, it is immediate that there is never any positive value of $\alpha$ which can satisfy (A.6). Suppose that $F'' < 0$ and consider a positive value of $\alpha$ further away from 0. Now instead of evaluating $F'$ at 0, we are evaluating $F'$ at $\alpha I_1 \geq 0$. Thus we must have $F'(\alpha I_1) \leq F'(0)$. If $F''(0) + s - 1 < 0$ it is necessarily true that $F'(\alpha I_1) + s - 1 < 0$. This implies that there are no positive values of $\alpha$ which can satisfy (A.6). Thus, if profits are decreasing in $\alpha$ at $\alpha = 0$, the firm must necessarily be at a corner solution in which it chooses to set $\alpha = 0$. As a result, the condition in Result 1 is both necessary and sufficient for the firm to choose $\alpha > 0$.

4. The firm produces $Q_x = Q_0$ with fixed contributions per unit sold

Here we will establish that when contributions are fixed per unit sold, firms which contribute to the public good will produce the same level of output of the private good as a firm which does not contribute. At a corner solution with $x = 0$, (18) will hold as a strict inequality. It will hold as an equality whenever $x > 0$. If we are at a corner solution with $x = 0$, $Q_x = Q_0$ will be the level of output which solves (17). (Compare (17) to (A.1) to verify that this is so.) If $x > 0$, then (18) holds as an equality implying that $F'' + x Q_x F'' - 1 = 0$. Substituting this into (17) again will yield $Q_x = Q_0$. As before the act of contributing to the public good has no effect on the firm’s choice of an optimal level of output.

5. Result 5

This section will demonstrate that contributions and net profits of the firm are higher when the firm optimally chooses $\alpha$, the percentage of profits to donated to charity compared to when it optimally chooses $x$, a fixed contribution per unit sold. This will presume that we are not at a corner solution of zero contributions in either model, so assume that $F'(0) > 1$. When fixed costs are positive, I will first show that contributions are higher when giving takes the form of a
percentage of profits. When fixed costs are zero, we will see that contributions are the same under the two mechanisms. Using equations (2), (6) and (11), note that the derivative in equation (10) may be expressed as follows:

\[
\frac{d \Pi_1^N}{d \alpha} = \left[ P_0 - C \right] Q_0 \left[ F' - 1 + \alpha \Pi_1 F'' + s(1 - \alpha F')^2 \right].
\]

(A.7)

The derivative in (A.7) shows the marginal effect on net profits from raising \( \alpha \), the share of profits donated to the public good. Let the value of \( x \) which solves (20) be denoted \( x^* \). When contributions are a percentage of profits they are given by \( \alpha \Pi_l \) and when they are fixed per unit sold they are given by \( x^*Q_0 \). When (20) holds as an equality we have \( F' - 1 + x^*Q_0 F''' = 0 \), where the arguments of \( F' \) and \( F''' \) are total contributions towards the public good. If \( \alpha \Pi_l = x^*Q_0 \) this will imply that \( F' - 1 + \alpha \Pi_l F''' = 0 \). Use this in (A.7) plus (11) to obtain the following:

\[
\frac{d \Pi_1^N}{d \alpha} = S \geq 0.
\]

(A.8)

If fixed costs \( S = 0 \) the condition in (A.8) will hold as an equality, and the same level of contributions will be made under both mechanisms. However, if \( S > 0 \), then \( d \Pi_1^N / d \alpha > 0 \) when the derivative is evaluated at \( \alpha \Pi_l = x^*Q_0 \). This implies that a firm donating a portion of its profits to the public good will optimally choose a value of \( \alpha \) such that total contributions are greater than \( x^*Q_0 \). Net profits are rising in \( \alpha \) when \( \alpha \Pi_l = x^*Q_0 \), so the firm will increase \( \alpha \) above the amount necessary to achieve this equation. If net profits are rising in \( \alpha \), then so are gross
profits $\Pi_i$. Since total contributions are $\alpha \Pi_i$, under an optimally chosen value of $\alpha$ we must have $\alpha \Pi_i > x*Q_0$.

Next, we will show that net profits are higher under an optimally chosen $\alpha$ compared with net profits under $x*$. The strategy is to show that when $\alpha \Pi_i = x*Q_0$, the firm donating a percentage of its profits will have the higher net profits. Because the derivative in (A.8) is positive at this point, when the optimal value of $\alpha$ is chosen, net profits will be even higher for the firm donating a percentage of its profits.

Since our starting point is $\alpha \Pi_i = x*Q_0$, using (4a) and (15), we have

$$\Pi_i - \Pi_x = 1 - [(P_x - C)Q_0 - S].$$

If $\Pi_i - \Pi_x > 0$, then when contributions are equal, the firm earns higher profits when donating a percentage of its profits towards the public good. If we substitute for $\Pi_1$ from (3), this becomes $\Pi_i - \Pi_x = (P_1 - P_x)Q_0$. Making use of (2), (5), and (16) yields $\Pi_i - \Pi_x = (\alpha(P_1 - C) - x*)F'Q_0$. (We can factor out $F'$ because $\alpha \Pi_i = x*Q_0$ means that the arguments of $F'$ are equal across the two mechanisms.) Because $\alpha \Pi_i = x*Q_0$, using (3) we can write $x* = \alpha(P_1 - C) - \alpha S / Q_0$. By substitution this yields $\Pi_i - \Pi_x = \alpha F'> 0$. Thus when contributions are equal and fixed costs are positive, the firm contributing a portion of its profits to charity earns higher net profits than the firm making a fixed contribution. When an optimal value of $\alpha$ is chosen, the gap in net profits will be even greater. This establishes the validity of the statements in Result 5.

**Part 2 - A Model of Contributions with Cournot Competition**

In this section of the appendix, I will show that the model’s results continue to hold when the firm’s competitors are made explicit in a model of Cournot oligopoly. Suppose the representative
consumer’s utility function is the following: \( U(Q^T) + F(G) \), where \( Q^T = \sum_{j=1}^{m} Q_j \) and \( m \) is the number of Cournot firms. All \( m \) firms have identical fixed costs \( S \) and marginal costs \( C \). All the private goods produced by the \( m \) firms are perfect substitutes. Let firm 1 be the firm making a donation to the public good, while firms \( j = 2, ..., m \) only sell the private good without making such a donation. When firm 1 chooses \( \alpha > 0 \), the expression for the price premium \( k \) that it can charge is same as in equation (6). It can be shown that at an interior equilibrium in which all \( m \) firms produce the private good, the following relationship among prices must hold:

\[
P_1 = \frac{P_j - \alpha F'(\alpha \Pi_1)C}{1 - \alpha F'(\alpha \Pi_1)}, \quad j = 2, ..., m,
\]

where as before, \( \alpha \) is the percentage of profits that firm 1 donates towards the public good. The price premium for firm 1 in (A.9) will leave the representative consumer indifferent between making a purchase from firm 1 and any of the other \( m \) Cournot competitors. Outside of the possible donation to the public good by firm 1, the firms are symmetric. Thus, all firms which do not contribute to the public good will charge the same price and sell the same quantity. In keeping with the notation of the main body of the paper, denote these as \( P_0 \) and \( Q_0 \) respectively.

Note that we can write \( P_0 = P_0(Q^T) \) reflecting the fact that the price is a function of the total number of units sold. By contrast, if firm 1 chooses a positive value for \( \alpha \), it will charge the price \( P_1 \) and sell the quantity \( Q_1 \).

For a firm which does not contribute towards the public good, the objective function, and first and second order conditions are as follows:

\[
\max_{\mathcal{C}_0} P_0(Q^T)Q_0 - C Q_0 - S, \quad (A.10)
\]
\[ P_0'(Q^T)Q_0 + P_0 - C = 0, \quad (A.11) \]
\[ P_0''(Q^T)Q_0 + 2P_0'(Q^T) < 0, \quad (A.12) \]

where \( P_0'(Q^T) < 0 \), and \( P_0''(Q^T) \) denote the first and second derivatives of the inverse demand function. In a Cournot model, it is standard to impose conditions which result in downward sloping reaction functions for the firms. This requires,

\[ P_0''(Q^T)Q_0 + P_0'(Q^T) < 0. \quad (A.13) \]

This condition implies that the marginal revenue of a firm is decreasing in the output of its rivals.

For firm 1, net profits are given by

\[ \Pi_1^N = (1-\alpha)\left(\frac{(P_0-C)Q_1}{1-\alpha F'_1(\Pi^G)} - S\right), \quad (A.14) \]

where the substitution for \( P_1 \) has been made from (A.9). The derivative of (A.14) plus the fact that the first order condition requires \( d\Pi_1^N / dQ_1 = 0 \) implies the following:\(^{27}\)

\[ \frac{(1-\alpha)}{1-\alpha F'_1(\Pi^G)}\left(P_0'(Q^T)Q_1 + P_0(Q^T) - C\right) = 0. \quad (A.15) \]

From equation (A.11), this equation will be satisfied when \( Q_1 = Q_0 \). Thus, firm 1 will produce the same level of output as the firms which do not contribute towards the public good. For all firms, the choice of \( Q \) is not affected by the choice of \( \alpha \). Thus, the analysis and results of section

\(^{27}\) It is helpful to note that \( \Pi^F = a\Pi^N/(1-\alpha) \), so that we can write \( F(a\Pi^N/(1-\alpha)) \). It simplifies the derivation of (A.15) to express \( F \) in this manner prior to taking the derivative of (A.14).
3.3 and 3.4 are unaffected by the consideration of Cournot competitors. The explicit consideration of competitors does not affect the results of the model because of the separability of the demand for public and private goods.

However, one advantage of making competition explicit is that we can derive the effect of an increase in the number of firms on \( s \), where \( s = S/[(P_0 - C)Q_0] \). Making use of the first order condition in (A.11) and the fact that \( Q^T = mQ_0 \) it is possible to show the following:

\[
\frac{dQ_0}{dm} = \frac{-\left(P_0'(Q^T) + QP_0''(Q^T)\right)Q_0}{P_0'(Q^T) + m(P_0'(Q^T) + Q_0P_0''(Q^T))} < 0, \text{ and} (A.16)
\]

\[
\frac{dQ^T}{dm} = \frac{m \frac{dQ_0}{dm} + Q_0 = \frac{P_0'(Q^T)Q_0}{P_0'(Q^T) + m(P_0'(Q^T) + Q_0P_0''(Q^T))}} > 0, \text{ (A.17)}
\]

where both expressions are signed by making use of (A.13). Since \( Q^T \) is increasing in \( m \) we know that \( P_0 \) is decreasing in the number of firms. Moreover, since \( Q_0 \) is declining in the number of firms, we must have \( ds/dm > 0 \) because \( (P_0 - C)Q_0 \) is decreasing in \( m \). Therefore an increase in the number of firms in the industry will lower profitability and raise \( s \).
References


Elfenbein, D. W., Fisman, R., McManus, B., 2012. Charity as a substitute for reputation:


